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2839 [1920, 274].

By translating the steps of the construction of a regular pentagon from plane geometry into algebra show that one of the fifth roots of unity is equal to

$$\frac{1}{4}(\sqrt{5}-1) + \frac{1}{4}i\sqrt{10+2\sqrt{5}}.$$

(This problem is proposed for solution in Wilczynski and Slaughter, *College Algebra with Applications*, Boston, 1916, p. 193.)

### I. SOLUTION BY H. L. OLSON, University of Michigan.

Consider a circle with center at the origin,  $O$ , and unit radius, intersecting the positive  $x$ -axis at  $A$  and the negative  $y$ -axis at  $D$ . With center at the mid-point,  $E$ , of  $OD$ , draw a circular arc passing through  $A$  and intersecting the positive  $y$ -axis at  $F$ . The radius of this circle is  $\frac{1}{2}\sqrt{5}$ , and hence  $\overline{OF} = \frac{1}{2}(\sqrt{5}-1)$ . With center  $A$  and radius  $AF$ , draw a circular arc intersecting the circle with center  $O$  at  $G$  and  $H$ .  $AG$  and  $AH$  are two sides of a regular pentagon inscribed in the circle  $O$ , and  $\overline{OG}$  and  $\overline{OH}$  are the Argand representations of two of the fifth roots of unity.

Since  $AF^2 = 1 + (\frac{1}{2}\sqrt{5} - \frac{1}{2})^2$ , the equations of the circles  $O$  and  $A$  are

$$\begin{aligned} x^2 + y^2 &= 1, \\ (x-1)^2 + y^2 &= 1 + (\tfrac{1}{2}\sqrt{5} - \tfrac{1}{2})^2. \end{aligned}$$

Hence, the coördinates of  $G$  and  $H$  are

$$x = \tfrac{1}{4}(\sqrt{5}-1), \quad y = \pm \tfrac{1}{4}\sqrt{10+2\sqrt{5}};$$

and two of the fifth roots of unity are  $\frac{1}{4}[(\sqrt{5}-1) \pm i\sqrt{10+2\sqrt{5}}]$ .

### II. SOLUTION BY OTTO DUNKEL, Washington University.

In texts on geometry a pentagon is usually constructed by dividing the radius  $OA$ , here taken as of unit length, at  $M$  in extreme and mean ratio. The length  $OM$  is laid off twice on the circle as chords giving the points  $A, K, G$ ; then  $AK$  is the side of a regular decagon and  $AG$  is the side of a regular pentagon. From the definition of extreme and mean ratio it follows that  $OM^2 = OA \cdot MA = 1 - OM$ , and hence  $OM = \frac{1}{2}(\sqrt{5}-1)$ . It easily follows that  $MG = OG = 1$  and hence the coördinates of  $G$  are  $x = \frac{1}{4}(\sqrt{5}-1)$ ,  $y = \sqrt{1-x^2} = \frac{1}{4}\sqrt{10+2\sqrt{5}}$ . Hence the complex number represented by  $G$  is as stated in the problem.

Also solved by T. M. BLAKSLEE, ARTHUR PELLETIER, A. V. RICHARDSON, C. H. RICHARDSON, and F. L. WILMER.

**2853 [1920, 377]. Proposed by J. S. BROWN, Southwest Texas State Normal College, San Marcos, Texas.**

Find the side and apothem of a regular pentagon inscribed in a circle without the use of extreme and mean ratio.

### THREE SOLUTIONS BY T. M. BLAKSLEE, Ames, Iowa.

I. Let  $x + iy$  be the point on the unit circle whose angle is  $36^\circ$ . Then the side of a regular inscribed pentagon will be  $p = 2y$ . In the equation  $(x + iy)^5 = -1$ , the coefficient of  $i$  is

$$y(5x^4 - 10x^2y^2 + y^4) = 0.$$

We wish the smaller of the two positive values of  $y$ . Therefore we can remove the factor  $y$  and if we substitute  $1 - y^2$  for  $x^2$ , our equation reduces to

$$16y^4 - 20y^2 + 5 = 0.$$

Whence  $y^2 = (10 - 2\sqrt{5})/16$ ,  $p = \frac{1}{2}\sqrt{10 - 2\sqrt{5}}$ , and the apothem is  $a = \frac{1}{4}(\sqrt{5} + 1)$ .